Bishop McGuinness Catholic High School
Calculus AB Summer Assignment 2024-2025
Instructor: Mr. Paul Smith, psmith@bmhs.us

For Calculus, you will need:

(1) A 1.5" binder, which needs to be separate from other classes
(2) A graphing calculator
(3) Mr. Smith will be putting together a workbook for this course that will be copied at a local printer. More information on this will be put forth in mid-July, but please expect that you will be billed in Facts no more than $50 for the purchase of this workbook. You will receive your copy at the start of the school year.

(Note: If you would like a college-level textbook, I recommend the following book:
Stewart, James. *Single Variable Calculus: Early Transcendentals.* 7th ed. Belmont, CA: Thomson Higher Education, 2012.)

(Note #2: Although I list my e-mail above, I may not be available at times during the summer, so please do not expect me to always respond instantly).

There are two parts to your summer assignment: a Delta Math review assignment and 3 labs / 1 video to watch (starting on p. 3).

(1) There will be a virtual summer packet on Delta Math that is designed to aid you in brushing up on foundational Precalculus skills needed to ensure your success in AP Calculus AB.

To sign up for Delta Math, follow these instructions:

A) Go to the following site: https://www.deltamath.com/students?code=5Y7W-M6WN (or you can go to www.deltamath.com, click "For Students" and then "Register" and enter the class code 5Y7W-M6WN). This is the same login for all classes.

(B) Click register with e-mail, input your e-mail (Bishop McGuinness e-mail, please) and then click "Check E-mail".

(C) You'll have to enter your name and a password (please, again, use your real name or else you won't be able to get credit).

(D) Once you've gotten this, you'll be able to log in. You should see the parts of the summer assignment posted there (Part 1 will be posted on 6/1/2024, Part 2 will be posted on 7/1/2024).

This virtual summer packet must be completed before school begins (due on 8/21/2024). Students who have difficulty with the skills on the summer packet should consider using resources available on Delta Math/Khan Academy/YouTube etc. to be properly prepared for your upcoming math course.

AP Calculus AB. Summer Assignment, Fall 2024.

Be prepared for an assessment on Pre Calculus topics within the first ten days of class. Please see list of things to know for this test below. I will spend limited time reviewing Pre Calculus material, so please come prepared to get into new Calculus material almost immediately. The sets of questions on Renweb will give you an idea of what will be on this initial assessment. The sets are estimated to take you approximately 4 hours in total to complete.

<u>Various things to know for the "Review" Test</u>

<u>Set #1</u>

Functions:
- Writing equations of lines
- Using rational exponents and radical form
- Identify and evaluate functions and state their domains.
- Use graphs of functions estimate function values, domains, ranges, intercepts.
- Identify even and odd functions.
- Determine intervals on which functions are increasing, decreasing, constant
- Determine maxima and minima of functions.
- Identify and graph parent functions and their transformations.
- Perform algebraic operations on functions and find compositions.
- Find inverse functions algebraically and graphically.

<u>Set #2</u>

Polynomials and Rationals:
- Divide polynomials using long division and synthetic division
- Find real and complex zeros of polynomial functions (and state multiplicity of each root)
- Graph polynomial functions
- Find end behavior of polynomials
- Analyze and graph rational functions (removable discontinuities aka "holes", vertical/horizontal asymptotes)
- Solve polynomial and rational inequalities

<u>Set #3</u>

Exponentials and Logarithms:
- Properties of exponentials and how to use them
- Evaluate, analyze, and graph exponential and logarithmic functions
- Converting between exponentials and logarithms
- Properties of logarithms and how to use them (change of base particularly)
- Solving exponential and logarithmic equations
- Exponential growth and decay

<u>Set #4</u>

Trigonometry:
- Major unit circle values
- Basic trigonometric identities (Pythagorean identities)
- Trigonometric graphs (sine, cosine, tangent, cotangent, secant, cosecant)
- Sum and difference formulas
- How to use inverse trig to find angle measures
- Solve trigonometric equations using algebraic techniques.

(2) You will be asked to complete 3 labs and one video set that will give you foundational knowledge for the first unit in Calculus. You are more than welcome to work with other people to finish these labs – they will be discussed in the first few days of class at the start of the school year.

Lab #1a: Rates of Change	**AP Topics:** 2.1, 2.3

Enduring Understanding: (CHA-2) Derivatives allow us to determine rates of change at an instant by applying limits to knowledge about rates of change over intervals and allow us to contrast that changeableness with the immutability of God.

Lab on Module #1a: Rates of Change

First, we define the terms "average rate of change" and "instantaneous rate of change."

a. An "average rate of change" (or AROC) on an interval $[x_0, x_1]$ is defined as $\frac{f(x_1)-f(x_0)}{x_1-x_0}$, or the change of the function over a certain interval.

b. The "instantaneous rate of change" (or IROC) at a point x_0 is defined as $\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$, or the rate of change at a certain point.

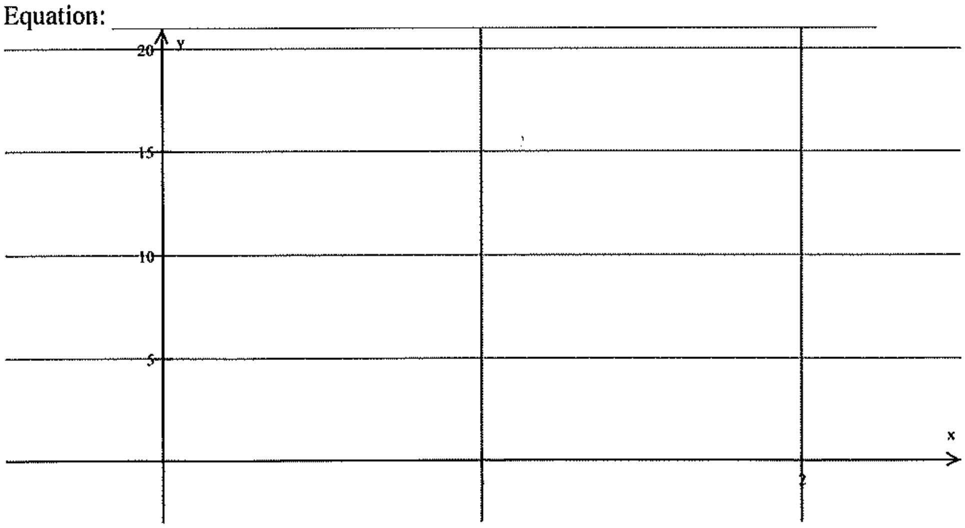
↖ as x gets closer to a

1. Each group should have a racquetball and a stopwatch (or another way of keeping time).
   a. One person in each pairing should be assigned to throw the ball, while the other person uses the stopwatch. The person throwing the ball should practice trying to get the ball as high as some measurable landmark before proceeding.
   b. When ready to record, the person with the stopwatch should record two times, using the "lap" feature on the stopwatch. When the ball reaches its peak, hit the lap button. When the ball first hits the ground, hit the stop button. This will give you both times recorded.
   c. Record your results, giving the height and time of the ball at three different times.

Time =	*Height =*
0 sec	
	0 ft

2. Use your graphing calculator to find a best fit quadratic model for the values found in the table above. Give the equation and provide a graph below.

Equation: ______________________________



3. Find the average velocity (change in distance over change in time) of the ball in the following time intervals using the model found in part (2).
   (a) $t = 0$ and $t = 2$: ________
   (b) $t = 0.9$ and $t = 1.0$: ________
   (c) $t = 0.99$ and $t = 1.0$: ________
   (d) $t = 0.999$ and $t = 1.0$: ________
   (e) $t = 1.0$ and $t = 1.001$: ________
   (f) $t = 1.0$ and $t = 1.01$: ________
   (g) $t = 1.0$ and $t = 1.1$: ________

4. Use all of the information above to try to determine the velocity of the ball at exactly 1 second. Explain how you arrived at your answer:

5. Use a similar process to estimate the velocity of the ball at exactly 0.5 seconds. Show the work that leads to your conclusion.

6. At what time was the instantaneous velocity of the ball equal to zero? Explain your reasoning.
   ____
   ____

Lab Reflection Questions

Name one connection that you were able to make between mathematical concepts in this lab.

Explain how this lab enabled you to go from a "real-life" observation to a general mathematical principle.

<u>Video Summary Sheet for Module #1b</u>
(access videos with video link in e-mail directed to Mr. Smith's You Tube page)

	Essential Knowledge	Examples	Personal Notes
4.1 4.3	*(CHA-3.A) Interpret the meaning of a derivative in context.* What is the acronym TAU used when describing a derivative in context? T A U How do you determine the units of derivative? What about the derivative of a derivative? Explain how you can tell from the graph of a function which derivative is larger.	*(CHA-3.C) Interpret rates of change in applied contexts.* The birth rate for LIPETS on an island is modeled by the function $B(t) = 100e^{0.04t}$ and the death rate for the same LIPET is given by $D(t) = 50e^{0.03t}$, where B and D are measured in LIPETS/year and t is measured in years. (A) Using correct units, find B(3) and interpret the meaning of this value in the context of the problem. (B) What is the rate of change of the LIPET population at time $t=3$? Suppose the function $D(t) = 12 + 6.1\cos\left(\frac{\pi}{6}t\right)$ models the depth, in feet, of the water t hours after 12 A.M. in a certain harbor. Find the instantaneous rate of change at 6 A.M. in feet per hour and explain how this IROC can be written symbolically in terms of D.	

Module #1b: Video Check Your Understanding

(1) The cost of producing T golden sombreros is $C = f(T)$ dollars. What do $f(2) = 10$ and $f'(2) = 3$ mean?

(2) The tide removes sand from a beach at a rate modeled by the function

$$R(t) = 3 + 5\sin\left(\frac{\pi}{12}t\right)$$

$R(t)$ is measured in cubic yards per hour and t is measured in hours for $0 \leq t \leq 12$. What is the rate at which the sand is being removed at time $t = 7$? Indicate units.

(3) (AP Classroom) A tire that is leaking air has an initial air pressure of 30 pounds per square inch (psi). The function $t = f(p)$ models the amount of time t, in hours, it takes for the air pressure of the tire to reach p psi. What are the units for $f'(p)$?

(4) Suppose the function $f(t)$ represents the rate at which people enter an amusement park (in people per hour) and $g(t)$ represents the rate at which people leave the amusement park (in people per hour), where t is measured in hours. Both functions are always positive. What statement would indicate that the rate of change of the number of people in the amusement park is increasing at time $t = k$? Explain your answer!

Answers to CYU: (1) $f(2)$ means \$10 to produce 2 sombreros; $f'(2)$ means \$3 more dollars to produce another sombrero when 2 have been produced

(2) 7.829 yd³/hr (3) hours per psi (4) $f'(k) > g'(k)$

Module #2a: Tangent Lines & Deriv. Graphs **AP Topics:** 2.2, 4.6 / 5.8, 5.9

Enduring Understandings: (CHA-3) Derivatives allow us to solve real-world problems involving rates of change and allow us to contrast that changeableness with the immutability of God.
(FUN-4) A function's derivative can be used to understand some behaviors of the function, an indication of God's providence through the order and efficient guidance of all things to a common end or purpose.

Lab on Module #2a: Tangent Lines

Here's the key question: can you find an easy way to approximate the value of a function with just simple adding, subtracting, multiplying and dividing? Let's take it back a moment...

(1) Graph the function $f(x) = \sqrt{x}$. Zoom in around $x = 9$. Zoom in again. And again. And again...what do you notice has happened to this function?

____

____

(2) You've discovered the concept of "local linearity"...the idea that certain types of functions can be approximated with linear functions if you "zoom in" far enough. Take two points near $x = 9$ and use the function values at those two points to approximate the IROC: ____________

(3) Then use the point-slope form to create a line that represents this function at $x = 9$:

____

(4) Why is this useful? If this function can be represented by a line at $x = 9$, would that make it easier to find something like, say, $\sqrt{9.1}$? Plug $x = 9.1$ into your line from (3). Then find the exact answer for $\sqrt{9.1}$. How do the two values relate?

____

____

(5) Can you follow the same procedure to estimate $\sqrt{51}$ using just a line? Two hints: make your numbers easier by centering your estimate at $x = 49$, and we will be able to show later that $f'(x) = \frac{1}{2\sqrt{x}}$. What did you get as your estimate using a line? Was it close?

____

____

Lab Reflection Questions

Name one connection that you were able to make between mathematical concepts in this lab.

Explain how this lab enabled you to go from a "real-life" observation to a general mathematical principle.

Module #2b: Basics of Accumulation — **AP Topics:** 6.1 / 6.2

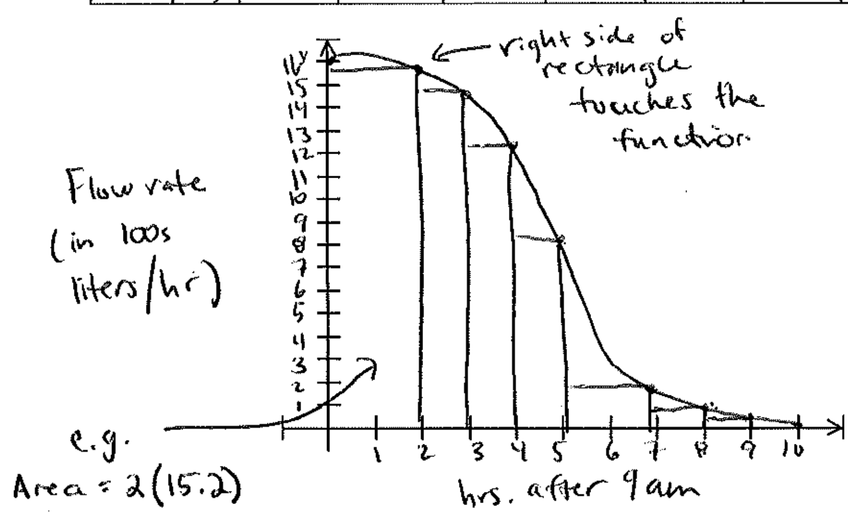
Enduring Understanding:
(CHA-4) Definite integrals allow us to solve problems involving the accumulation of change over an interval and allow us to recognize that accumulation as an indicator of God's transcendence of spatial limitations.

<u>Lab on Module #2b: Basics of Accumulation</u>

1. The problem we will investigate for the next couple of weeks is this: "Find the area of the region that lies under a function from *a* to *b*." Let's start with a motivating problem.

Yesterday, an oil tanker collided with a Coast Guard cutter off the California coast at 9:00 am, and the disabled tanker began to spill oil from its damaged hull. The rate of flow of oil into the Pacific Ocean was measured at several different times. The rates are listed in the table below.

Time	9:00	11:00	Noon	1:00	2:00	4:00	5:00	6:00	7:00
Flow rate (in 100s liters/hr)	16	15.2	14.4	12	8	1.2	0.9	0.4	0



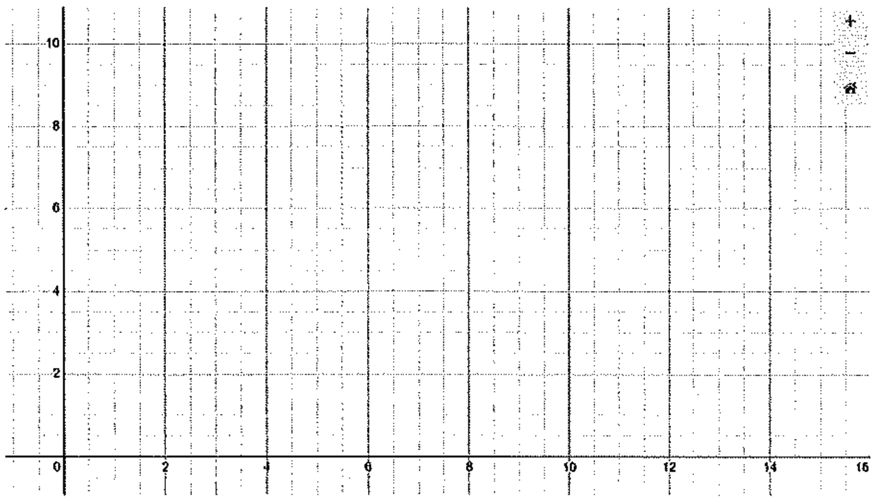
We will use areas of rectangles (a "known" shape) to approximate the "area under a curve." The use of an infinite number of rectangles gives us what is called a "Riemann Sum." What you see outlined above is a "right rectangle approximation method," (RRAM) where the right side of each rectangle touches the function at the coordinates listed. While this isn't a perfect method, it is pretty <u>close</u> to accurate. ***By finding the areas of each of the rectangles, give an approximation for the "area under the curve." Show your work.***

Using a different color pen or colored pencil, draw a set of rectangles where the <u>left</u> side of each rectangle (LRAM) touches the function at the coordinates listed. ***Then approximate the area underneath the curve by finding the area of all of the LRAM rectangles you've created.***

2. Consider the table of a continuous function $f(x)$ below:

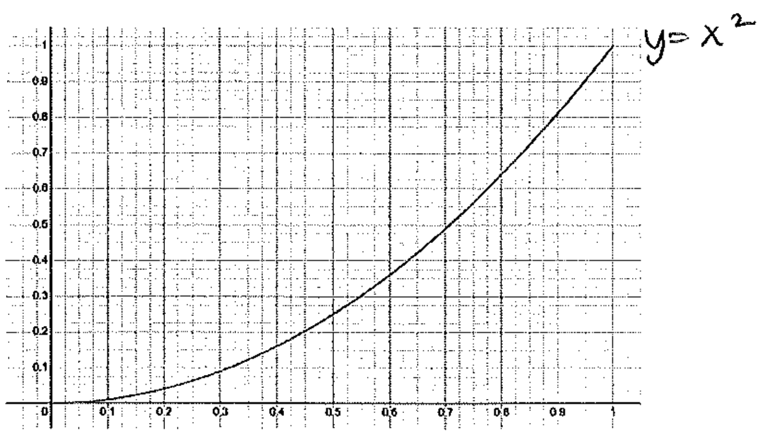
x	0	2	3	7	9	10	12	13	15
$f(x)$	3	6	7	6	8	10	4	3	8

Plot the points of the function below. Then use the "left" and "right" rectangle approximations to estimate the area underneath $f(x)$ from $x = 0$ to $x = 15$.



3. Consider the function $f(x) = x^2$. We are going to try to find the area underneath this curve on the interval [0, 1]. Draw 10 rectangles of equal bases where (1) the left endpoint of each rectangle touches the function (2) the right endpoint of each rectangle touches the function (3) the height of the rectangle is the height of the function at the midpoint. In each case, we have created 10 rectangles of equal width on the interval [0, 1].

Find the areas of each of these rectangles by finding the width of each of the rectangles and multiplying those widths by the heights (the height is the function value at the x-coordinate of the endpoint).



4. Your calculator has the capability of using many more rectangles than you are probably willing to use. The number of rectangles used by the calculator is so many that it almost approaches infinity (and thus gets close to being a Riemann sum). Go to MATH on your calculator and find #9:fnInt. Type $\int_0^1 x^2dx$ (or fnInt(x^2, x, 0, 1) for an older calculator) and use it to approximate the area underneath $f(x) = x^2$ from [0,1]. How does your answer compare to your approximations?