Bishop McGuinness Catholic High School
Calculus BC Summer Assignment 2024-2025
Instructor: Mr. Paul Smith, psmith@bmhs.us

For Calculus, you will need:

1. A 2" binder, which needs to be separate from other classes
2. A graphing calculator

(Note: If you would like a college-level textbook, I recommend the following book:
Stewart, James. *Single Variable Calculus: Early Transcendentals.* 7th ed. Belmont, CA: Thomson Higher Education, 2012.)
(Note #2: Although I list my e-mail above, I may not be available at times during the summer, so please do not expect me to always respond instantly).

There are two parts to your summer assignment: (1) a Delta Math assignment and (2) completion of the first couple of modules related to limits.

(1) There will be a virtual summer packet on Delta Math that is designed to aid you in brushing up on foundational Precalculus skills needed to ensure your success in AP Calculus BC.

To sign up for Delta Math, follow these instructions:

A) Go to the following site: https://www.deltamath.com/students?code=P33A-S2WP (or you can go to www.deltamath.com, click "For Students" and then "Register" and enter the class code P33A-S2WP). This is the same login for all classes.

(B) Click register with e-mail, input your e-mail (Bishop McGuinness e-mail, please) and then click "Check E-mail".

(C) You'll have to enter your name and a password (please, again, use your real name or else you won't be able to get credit).

(D) Once you've gotten this, you'll be able to log in. You should see the parts of the summer assignment posted there (Part 1 will be posted on 6/1/2024, Part 2 will be posted on 7/1/2024).

This virtual summer packet must be completed before school begins (due on 8/21/2024). Students who have difficulty with the skills on the summer packet should consider using resources available on Delta Math/Khan Academy/YouTube etc. to be properly prepared for your upcoming math course.

AP Calculus BC. Summer Assignment. Fall 2024.

Be prepared for an assessment on Pre Calculus topics within the first ten days of class. Please see list of things to know for this test below. I will spend limited time reviewing Pre Calculus material, so please come prepared to get into new Calculus material almost immediately. The sets of questions on Renweb will give you an idea of what will be on this initial assessment. The sets are estimated to take you approximately 4 hours in total to complete.

Various things to know for the "Review" Test

Set #1

Functions:
- Writing equations of lines
- Using rational exponents and radical form
- Identify and evaluate functions and state their domains.
- Use graphs of functions estimate function values, domains, ranges, intercepts.
- Identify even and odd functions.
- Determine intervals on which functions are increasing, decreasing, constant
- Determine maxima and minima of functions.
- Identify and graph parent functions and their transformations.
- Perform algebraic operations on functions and find compositions.
- Find inverse functions algebraically and graphically.

Set #2

Polynomials and Rationals:
- Divide polynomials using long division and synthetic division
- Find real and complex zeros of polynomial functions (and state multiplicity of each root)
- Graph polynomial functions
- Find end behavior of polynomials
- Analyze and graph rational functions (removable discontinuities aka "holes", vertical/horizontal asymptotes)
- Solve polynomial and rational inequalities

Set #3

Exponentials and Logarithms:
- Properties of exponentials and how to use them
- Evaluate, analyze, and graph exponential and logarithmic functions
- Converting between exponentials and logarithms
- Properties of logarithms and how to use them (change of base particularly)
- Solving exponential and logarithmic equations
- Exponential growth and decay

Set #4

Trigonometry:
- Major unit circle values
- Basic trigonometric identities (Pythagorean identities)
- Trigonometric graphs (sine, cosine, tangent, cotangent, secant, cosecant)
- Sum and difference formulas
- How to use inverse trig to find angle measures
- Solve trigonometric equations using algebraic techniques.

(2) You will be asked to complete the first part of Unit 1 that will give you foundational knowledge. You are more than welcome to work with other people to finish these notes – they will be discussed in the first few days of class at the start of the school year. (See the following pages….)

Module #4: Basics of Limits & Algebra of Limits **AP Topics: 1.2 – 1.9**

Enduring Understanding: (LIM-1) Reasoning with definitions, theorems, and properties can be used to justify claims about limits, suggesting the omnipresence of God as the immanent cause of all and unable to be circumscribed, measured or divided by any spatial relations.

Lab on Module #4: Limits

The limit is the "building block" of Calculus. Our simple definition of a limit in Precalculus was "The limit is the intended output of the function." Another way of saying this is:

$\lim\limits_{x \to a} f(x) = L$ means as x gets closer to a, $f(x)$ gets closer to L.

In other words, we are limiting the possibilities for values of the function to one possibility. We often use limits (often infinite limits) in everyday thought and conversation.

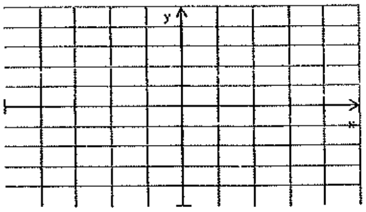
- As time goes on, a person's height levels off.
- As you drive on an on-ramp to a highway, your speed gets closer and closer to the highway's speed limit.
- With a pot of water on a heated stove, the water's temperature gets closer to boiling point as time goes on.

Limits can represent two concepts: short-term intentions and long-term intentions.

1. Sketch the following functions and then briefly explain why you think the specified limits do not exist. Your answer should be more specific than "there is no intended height for the function"—explain why there is no "intended" height at that x-value.

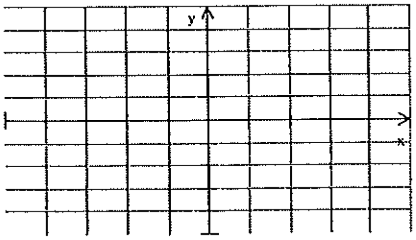
Limit #1: $\lim\limits_{x \to 1-} \frac{x|1-x|}{1-x} =$ _____

$\lim\limits_{x \to 1+} \frac{x|1-x|}{1-x} =$ _____



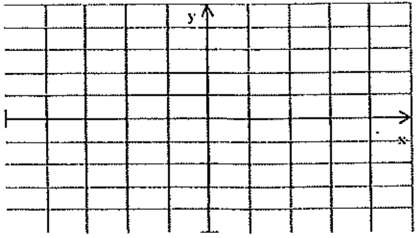
$\lim\limits_{x \to 1} \frac{x|1-x|}{1-x}$ DNE because: ____________________

Limit #2: $\lim\limits_{x \to 0} \sin\left(\frac{1}{x}\right) =$ _____



DNE because: ____________________

Limit #3: $\lim\limits_{x \to 2} \frac{1}{x-2}$

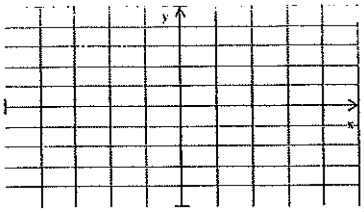


DNE because: ____

2. Suppose a table of values for the function $g(x)$ is given below. What is the best conclusion for the value of $\lim_{x \to 6+} g(x)$? ______ $\lim_{x \to 6-} g(x)$? ______ $\lim_{x \to 6} g(x)$? ______

x	5.97	5.98	5.99	6.00	6.001	6.01	6.02	6.03
$g(x)$	15.16	15.09	15.02	18	18.01	18.03	18.18	18.23

3. Consider $f(x) = \frac{x^2-5x+6}{x-2}$. Graph the function below. Then complete the accompanying tables of values:



x	0.9	0.99	0.999	1	1.001	1.01	1.1
$f(x)$							

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$							

$\lim_{x \to 1} f(x) =$ ____________ $\lim_{x \to 2} f(x) =$ ________________

How do those limit values compare to $f(1)$ and $f(2)$?

____

4. Suppose that $f(x) = \begin{cases} 2x+1, x < 1 \\ e^x + 3, x > 1 \end{cases}$.

Then $\lim_{x \to 1-} f(x) =$ ________ and $\lim_{x \to 1+} f(x) =$ __________

Lab Reflection Questions

Name one connection that you were able to make between mathematical concepts in this lab.

If the one-sided limits (to the left and right of an input value) are not equal, does the limit exist? Think about the "intended" height concept when formulating your answer.

Video Summary Sheet for Module #4a

1.2
1.3
1.4
1.5

Essential Knowledge

(LIM-1.A, LIM-1.B) Represent and interpret limits analytically using correct notation.
State the formal definition of a limit below and explain that definition using this graph.

____
____
____

(LIM-1.C) Estimate limits of functions.
What is difference in $\lim\limits_{x\to 6-} f(x)$ vs. $\lim\limits_{x\to 6+} f(x)$?

____
____

What are 3 conditions that might cause a limit to not exist (DNE)?

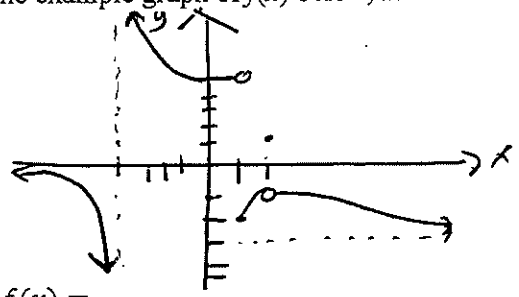
(1) ______________________________
(2) ______________________________
(3) ______________________________

Suppose $\lim\limits_{x\to a} f(x) = L$ and $\lim\limits_{x\to a} g(x) = M$. Then

(1) $\lim\limits_{x\to a}[f(x) \pm g(x)] =$ ____________
(2) $\lim\limits_{x\to a}[cf(x)] =$ ____________
(3) $\lim\limits_{x\to a}[f(x)g(x)] =$ ____________
(4) $\lim\limits_{x\to a}\left[\frac{f(x)}{g(x)}\right] =$ ____________
(if M is not ______)
(5) $\lim\limits_{x\to a} c =$ ____________
(6) $\lim\limits_{x\to a} x^n =$ ____________

Examples

For the example graph of *f*(*x*) below, find the following limits.



$\lim\limits_{x\to\infty} f(x) =$ ____________
$\lim\limits_{x\to -4-} f(x) =$ ____________
$\lim\limits_{x\to 1+} f(x) =$ ____________
$\lim\limits_{x\to 2} f(x) =$ ____________

Create a table of values for $g(x) = \frac{\sqrt{x+1}-2}{x-3}$ below that shows $\lim\limits_{x\to 3} g(x) = \frac{1}{4}$.

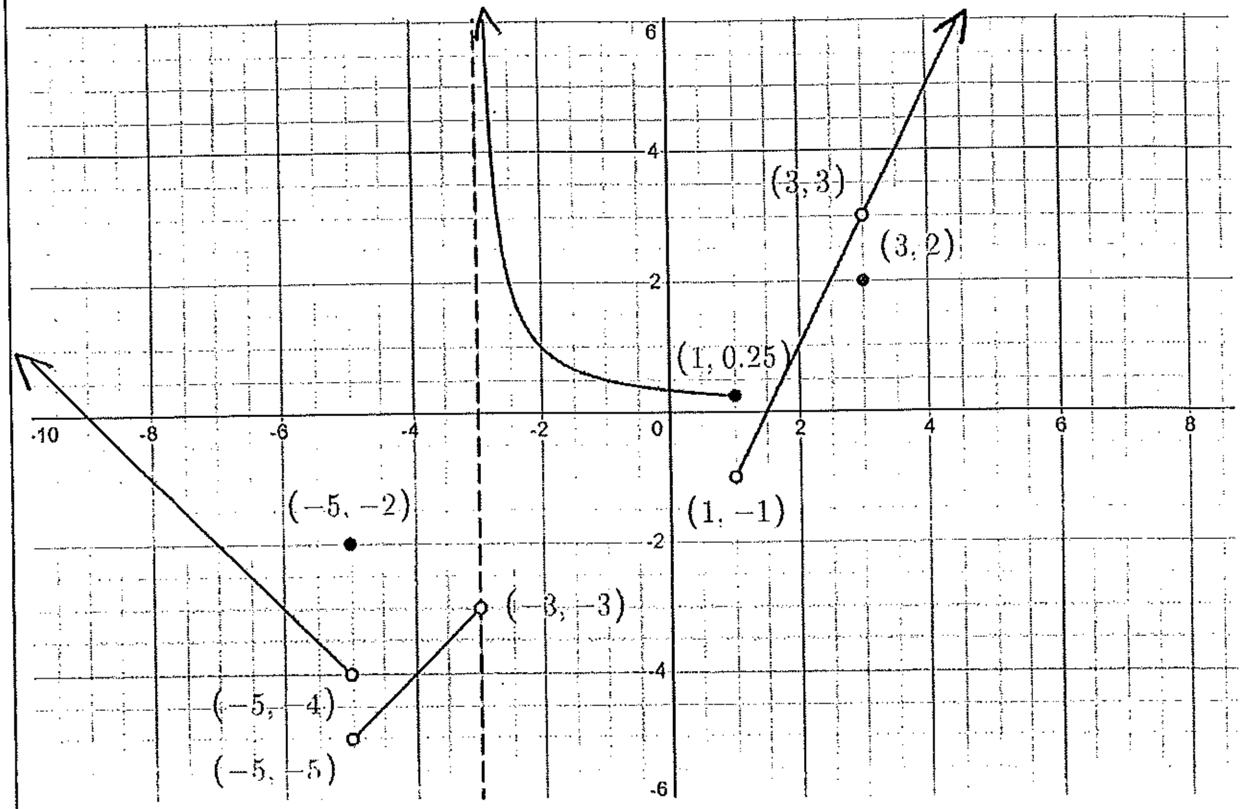
x							
$g(x)$							

(LIM-1.D) Determine the limits of functions using limit theorems.
Suppose $\lim\limits_{x\to 3} f(x) = 5$ and $\lim\limits_{x\to 3} g(x) = 4$. Find...

(A) $\lim\limits_{x\to 3}[7f(x)] =$ ____________
(B) $\lim\limits_{x\to 3}\left[\frac{g(x)}{f(x)} - x^2 + 5\right] =$ ____________
(C) $\lim\limits_{x\to 3+}\left[\frac{f(x)}{12} * \sqrt{g(x)}\right] =$ ____________

Personal Notes

Module #4a: Video Check Your Understanding



The figure above shows the graph of the function *f*. Use the graph to answer #1 – 3.

1. Which of the following statements about *f* are true?
   A. $\lim_{x\to -3-} f(x) = -3$
   B. $\lim_{x\to 1-} f(x) = f(1)$
   C. $\lim_{x\to 1-} f(x) = \lim_{x\to 1+} f(x)$
2. What is $\lim_{x\to 3} f(x)$?
3. What is $(\lim_{x\to -5-} f(x))(\lim_{x\to -5+} f(x))$?

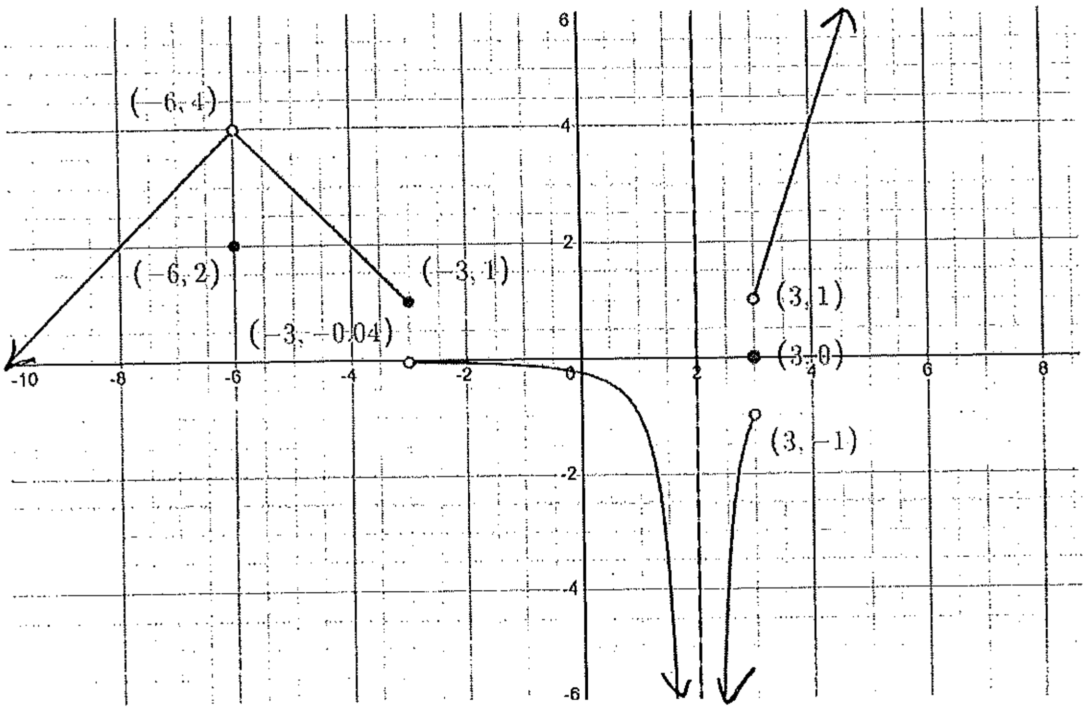
$f(2) = 1$	$\lim_{x\to 2} f(x) = -1$
$g(2) = -3$	$\lim_{x\to 2} g(x) = 8$
$h(4) = 5$	$\lim_{x\to 4} h(x) = 5$

4. The table above gives selected values and limits of the functions *f*, *g*, and *h*. What is $\lim_{x\to 2}(2f(x) + g(x))h(x^2)$?

Answers to CYU:

(1) A and B only (2) 3 (3) 20 (4) 30

Lesson 4a Examples



The figure above shows the graph of the function *f*. Use the graph to answer #1 – 5.

1. Which of the following statements about *f* are true?
   A. $\lim_{x\to -6} f(x) = f(-6)$
   B. $\lim_{x\to 3-} f(x) = \lim_{x\to 3+} f(x)$
   C. $\lim_{x\to -3-} f(x) = f(-3)$

2. What is $\lim_{x\to -6} f(x)$?

3. What is $\lim_{x\to 2-} f(x)$?

4. Which of the following limits does not exist?
   A. $\lim_{x\to -6} f(x)$
   B. $\lim_{x\to 3-} f(x)$
   C. $\lim_{x\to 3} f(x)$

5. Suppose $\lim_{x\to -6} h(x) = 3$. What is $\lim_{x\to -6}\left(2f(x)h(x)\right)$?

Lesson 4a Examples

6.

$f(-5) = 6$	$\lim_{x\to-5} f(x) = -5$
$g(-5) = -6$	$\lim_{x\to-5} g(x) = -6$
$h(-5) = -7$	$\lim_{x\to-5} h(x) = -2$

The table above gives selected values and limits of the functions *f*, *g*, and *h*. What is $\lim_{x\to-5}\frac{(5f(x)-7g(x))}{h(x)}$?

7. Suppose a table of values for the function *g*(*x*) is given below. What is the best conclusion for the value of $\lim_{x\to8} g(x)$?

x	7.86	7.91	7.96	7.99	8.01	8.04	8.10	8.16
g(*x*)	15.86	15.91	15.96	15.99	16.01	16.04	16.10	16.16

8. Suppose a table of values for the function *g*(*x*) is given below. What is the best conclusion for the value of $\lim_{x\to-1+} f(x)$? $\lim_{x\to-1-} f(x)$? $\lim_{x\to-1} f(x)$?

x	-1.05	-1.01	-1.004	-1.001	-0.997	-0.98	-0.97	-0.95
f(*x*)	400	10000	62500	1000000	4.994	4.960	4.940	4.900

Video Summary Sheet for Module #4b

	Essential Knowledge	Examples	Personal Notes
1.6 1.7 1.8 1.9	*(LIM-1.E) Determine the limits of equivalent expressions for the function or the Squeeze Theorem.* What should you always do first when confronted with a limit to be evaluated using an equation? ____________ ____________ Name some techniques that are helpful for evaluating limits: (1) ____________ (2) ____________ (3) ____________ Explain the Squeeze Theorem in your words and use the example of $\lim_{x\to 0} x^2 \cos\left(\frac{1}{x}\right)$ to demonstrate (include a graph as part of your demonstration).	Find $\lim_{t\to 2}\frac{t^3+3t^2-12t+4}{t^3-4t}$, showing your work below. For $h(t)$ below: $h(t)=\begin{cases}\frac{3\lvert t+1\rvert}{t+1}, & t<-1\\ 0, t=-1 & \\ t^2-t-9, & t>-1\end{cases}$ $\lim_{t\to -1+} h(t) =$ ____ $\lim_{t\to -1-} h(t) =$ ____ $\lim_{t\to -1} h(t) =$ ____ Find $\lim_{x\to 1}\frac{\sqrt{x}-1}{x^2-1}$ and show your work. What are the following two key limits? (A) $\lim_{x\to 0}\frac{\sin(x)}{x} =$ (B) $\lim_{x\to 0}\frac{1-\cos(x)}{x} =$	

Module #4b: Video Check Your Understanding

Evaluate the following limits. Show your work as indicated on the video.

(1) $\lim\limits_{x \to -4} \frac{x^2+5x+4}{x^2+3x-4} =$

(2) $\lim\limits_{x \to -4} \frac{x^3+7x^2+2x-40}{x+4} =$

(3) If $3x \le f(x) \le x^2 - 4$ for $3 < x < 5$, what is $\lim\limits_{x \to 4} f(x)$? Explain your reasoning.

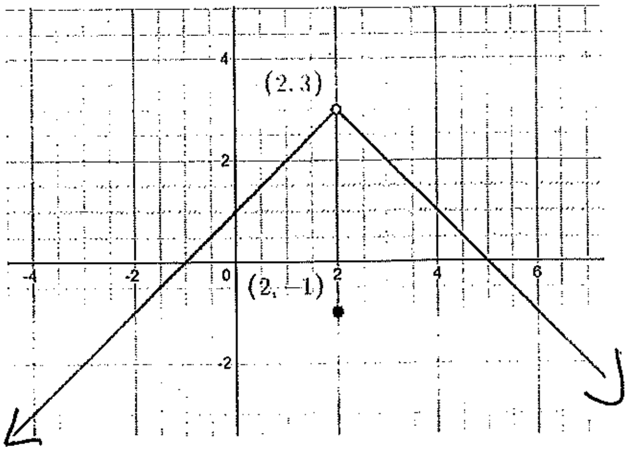
(4) If h is a piecewise linear function such that $\lim\limits_{x \to 2} h(x)$ does not exist, which of the following could represent the function h?

(A) $h(x) = \begin{cases} 6 - x^2, x < 2 \\ undefined, x = 2 \\ 2x - 2, x > 2 \end{cases}$

(B)

x	1.8	1.91	1.95	1.99	2	2.01	2.1	2.15	2.2
$h(x)$	6.8	6.92	6.95	6.98	7	1.2	1.14	1.11	1.08

(C)



Answers to CYU:

(1) 3/5 (2) -6 (3)12 (4) B only

Lesson 4b Examples

(1) $\lim\limits_{x\to4}\frac{x^2-4x}{x^2-3x-4}=$

(2) $\lim\limits_{x\to2}\frac{x^3-4x}{x-2}=$

(3) Let g and h be the functions defined by $g(x)=\sin\left(\frac{\pi}{2}x\right)+4$ and $h(x)=\frac{1}{3}x^2-\frac{2}{3}x+\frac{16}{3}$. If f is a function that satisfies $g(x)\le f(x)\le h(x)$ for $0<x<2$, what is $\lim\limits_{x\to1}f(x)$?

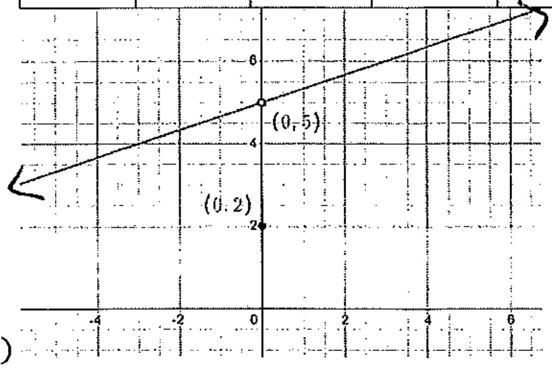
(4) If g is a piecewise linear function such that $\lim\limits_{x\to0}g(x)=5$, which of the following could represent the function g?

(A) $g(x)=\begin{cases}-\frac{x^2-5x}{x} & for\ x\neq0\\ 2\ for\ x=0\end{cases}$

(B)

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1	0.2
g(x)	200	2000	20000	5	-20000	-2000	-200	-100

(C)



Lesson 4b Examples

(5) $\lim_{x \to 10} \frac{|x-10|}{10-x} =$

(6) It can be shown that $\lim_{x \to 0} \frac{\sin(3x)}{3x} = 1$. Find $\lim_{x \to 0} \frac{1-cos^2(3x)}{3x}$.

(7) $\lim_{t \to 9} \frac{9-t}{3-\sqrt{t}} =$

(8) Suppose $f(x) = \begin{cases} 5x - 4 \; for \; x < 1 \\ \sin\left(\frac{\pi x}{2}\right) for \; x \geq 1 \end{cases}$. Then $\lim_{x \to 1-} f(x) =$

Module #5: Asymptotes & Continuity I — **AP Topics: 1.14, 1.15 / 1.11, 1.12**

Enduring Understanding: (LIM-1) Reasoning with definitions, theorems, and properties can be used to justify claims about limits, pointing to the eternal nature of God as beyond temporal relations with neither beginning nor end.

(LIM-2) Reasoning with definitions, theorems, and properties can be used to justify claims about continuity, an indication of God's providence through the order and efficient guidance of all things to a common end or purpose.

Lab on Module #5: Asymptotes & Continuity I

Two concepts will allow us to expand our ability to describe functions and graphs.

- We have an infinite limit if any of the following is true:
    - $\lim_{x\to a+} f(x) = \infty$ (f increases without bound)
    - $\lim_{x\to a-} f(x) = \infty$ (f increases without bound)
    - $\lim_{x\to a+} f(x) = -\infty$ (f decreases without bound)
    - $\lim_{x\to a-} f(x) = -\infty$ (f decreases without bound)
- We have a limit at infinity if either of the following are true:
    - $\lim_{x\to\infty} f(x) = L$
    - $\lim_{x\to-\infty} f(x) = L$

As the outputs become unbounded, we get a "vertical asymptote" with equation $x = a$. In other words, $f(x) \to \infty$ or $f(x) \to -\infty$ as x approaches a from the left or right.

As the inputs become unbounded, we get a horizontal asymptote of $y = L$. This could mean either:

$\lim_{x\to\infty} f(x) = L$ or $\lim_{x\to-\infty} f(x) = L.$

Here, we will use graphs and tables to investigate patterns in vertical and horizontal asymptotes. Use limit statements to complete your justification.

1. What are the vertical asymptotes for $f(x) = \frac{1}{x^2-1}$? Fill out the chart to find out, and then conclude with limit statements.

x	-0.98	-0.99	-0.999	-1.001	-1.01	-1.1
$f(x)$						

x	0.98	0.99	0.999	1.001	1.01	1.1
$f(x)$						

f has a vertical asymptote at $x =$ _______ because of these limits: ____________________

f has a vertical asymptote at $x =$ _______ because of these limits: ____________________

2. What are the vertical asymptotes for $f(x) = \frac{1}{x^3-x^2-x+1}$? Fill out the chart to find out, and then conclude with limit statements.

x	-0.98	-0.99	-0.999	-1.001	-1.01	-1.1
$f(x)$						

x	0.98	0.99	0.999	1.001	1.01	1.1
$f(x)$						

f has a vertical asymptote at $x=$ ________ because of these limits: ____________________

f has a vertical asymptote at $x=$ ________ because of these limits: ____________________

3. What are the horizontal asymptotes for $f(x) = \frac{2x^2+1}{3x^2+x}$? Fill out the chart to find out, then conclude with limit statements.

x	-10000	-1000	-100	100	1000	10000
$f(x)$						

4. What are the horizontal asymptotes for $f(x) = \frac{2x^2+e^x}{3x^3+6e^x}$? Fill out the chart to find out, then conclude with limit statements.

x	-10000	-1000	-100	100	1000	10000
$f(x)$						

5. What are the horizontal asymptotes for $f(x) = \frac{(x-2)|x|}{5x^2+x}$? Fill out the chart to find out, then conclude with limit statements.

x	-10000	-1000	-100	100	1000	10000
$f(x)$						

Lab Reflection Questions

Name one connection that you were able to make between mathematical concepts in this lab.

Explain how this lab enabled you to go from a "real-life" observation to a general mathematical principle.

AP Calculus AB. Module #5: Asymptotes & Continuity I. Fall 2023.

Video Summary Sheet for Module #5a

	Essential Knowledge	Examples	Personal Notes
1.14 1.15	*(LIM-2.D) Interpret the behavior of functions using limits involving infinity.* Using limits, explain what is required to have an infinite discontinuity. Make sure to connect with the term "asymptote". ________________ ________________ ________________ What are two possible limits would indicate that a function has a horizontal asymptote? 1. ____________ 2. ____________ What's the order for the "Power Tower"?	Write a limit statement that would mean a horizontal asymptote of $y = -2$ for a function $f(x)$ and a limit statement for a vertical asymptote of $x = 3$ for $f(x)$. Explain why $f(x) = \frac{x^2-16x+55}{x^2-13x+40}$ has a vertical asymptote at $x = 8$. Explain why $\lim\limits_{x\to\infty} \frac{5x^8-7}{6+4x^8} = \frac{5}{4}$ Show how to find $\lim\limits_{x\to\infty} \frac{7-x^4}{x^3+2^x}$ and $\lim\limits_{x\to\infty} \frac{2(7^x)+5}{x^8+3(7^x)} = \frac{2}{3}$ Explain how to find the H.A. for $y = \frac{3+2e^x}{e^x-5}$	

Module #5a: Video Check Your Understanding

(1) Suppose we have the following limits for a function f(x):

$$\lim_{x \to 3^-} f(x) = \infty$$
$$\lim_{x \to 4^+} f(x) = -\infty$$
$$\lim_{x \to -\infty} f(x) = 9$$
$$\lim_{x \to \infty} f(x) = 5$$

List the equations of all of the vertical asymptotes and horizontal asymptotes for f(x).

(2) $\lim\limits_{x \to \infty} \frac{4x^4+5}{(x^2-2)(2x-1)^2} =$

(3) Find the vertical asymptote(s) (if any) for $f(x) = \frac{2x-6}{(x-3)(x+4)}$. Use limits to explain your answer.

(4) Find the horizontal asymptote(s) for $f(x) = \frac{2x^2+x-1}{x^4+x-2}$. Use limits to explain your answers.

Answers to CYU: (1) Vertical asymptotes: $x = 3$, $x = 4$;horizontal asymptotes: $y = 5$, $y = 9$

(2) 1 (3) $x = -4$ ($x = 3$ is a hole) (4) $y = 0$