Due 8/18/22

Bishop McGuinness Catholic High School
Calculus AB Summer Assignment
Instructor: Mr. Paul Smith, psmith@bmhs.us

For Calculus, you will need:

(1) A 2" binder, which needs to be separate from other classes
(2) A graphing calculator
(Note: no book will be required for this course. However, I can offer book suggestions if you would like an additional resource).
(Note #2: Although I list my e-mail above, I may not be available at times during the summer, so please do not expect me to always respond instantly).

There are two parts to your summer assignment.

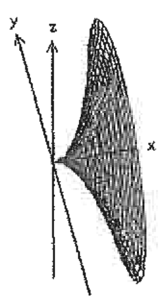
(1) **Type your response to the following questions. These responses will count as your first journal entries for the course. If you use outside materials, make sure to cite your sources.** Please put the date of your journal entry on the top right of the page and the title of the journal entry on the top line. The title of this journal entry is "Calculus AB Summer Assignment."

Personal Mathematical Questions

1. Now that you have been through Pre Calculus, what is <u>your</u> definition of mathematics? How would characterize your experience in mathematics last year? (Your definition needs to incorporate your experience from last year).
2. Are you good in math? Explain your reasoning.

Driving Calculus Questions

3. Find a way to approximate $\sqrt{61}$ without a calculator. You should be able to extend your reasoning to find $\sqrt{a}$ for any positive integer $a < 100$.
4. Consider the shape that is formed when the area between the x-axis and $y = x^2$ from $x = 0$ to $x = 4$ is rotated around the x-axis (see figure below). What is the volume of this shape? Be able to explain your reasoning.



5. What is the length of the orbit of the Earth when going around the Sun? Show the work or explain the reasoning that leads to your answer.

(2) **Be prepared for an assessment on Pre Calculus topics within the first ten days of class.** Please see list of things to know for this test below. I will spend limited time reviewing Pre Calculus material, so please come prepared to get into new Calculus material almost immediately. The sets of questions below will give you an idea of what will be on this initial assessment. I will count your completion of these problems towards a grade at the beginning of the year. You may also turn in these problems on the day of your assessment and I will spot check random parts to give some extra credit.

<u>Various things to know for the "Review" Test</u>

<u>Set #1</u>

Functions:
- Writing equations of lines
- Using rational exponents and radical form
- Identify and evaluate functions and state their domains.
- Use graphs of functions estimate function values, domains, ranges, intercepts.
- Identify even and odd functions.
- Determine intervals on which functions are increasing, decreasing, constant
- Determine maxima and minima of functions.
- Identify and graph parent functions and their transformations.
- Perform algebraic operations on functions and find compositions.
- Find inverse functions algebraically and graphically.

1. Find an equation of a line in point-slope form having the given characteristics:
   (a) Slope = -3, contains the point (2, -4)
   (b) Slope = 0, contains the point (-5, 4)
   (c) Contains the points (3, -4) and (2, 1)
   (d) Parallel to the line $2x - 3y = -4$, contains the point (-5, 3)
   (e) Perpendicular to the line $x + y = 2$, contains the point (4, -3)

2. Simplify the following expressions:
   (a) $\sqrt[5]{a^{10}b^7}$ (b) $\sqrt{16t^8u^{16}}$ (c) $\frac{y^{\frac{3}{4}}y^{\frac{2}{3}}}{y^{\frac{5}{12}}}$ (d) $\frac{16^{\frac{3}{4}}}{16^{\frac{1}{4}}}$

   Rewrite the following using rational exponents. Example: $\frac{1}{\sqrt[3]{x^2}} = x^{-\frac{2}{3}}$
   (a) $\sqrt[5]{x^3} + \sqrt{x+1}$ (b) $\frac{1}{\sqrt{x}} - \frac{2}{x}$ (c) $\frac{1}{\sqrt[3]{x^2}} + \frac{1}{4x}$ (d) $\frac{1}{4\sqrt{x}} - \frac{1}{3x^2}$

3. Find the following for each given function:
   (i) $f(-2)$, $f(-x)$ and $f(x-2)$
   (ii) The domain and range of the function.
   (iii) Determine whether the function is even, odd, or neither
   (iv) Determine intervals on which the function is increasing, decreasing, constant
   (v) Determine maxima and minima of the function.

   (a) $f(x) = \sqrt{x^2 - 4}$ (b) $g(x) = \frac{x^3}{x^2-9}$ (c) $h(x) = x^3 - 4x$
   (d) $p(x) = \begin{cases} 2.5x + 11, x \leq -2 \\ 0.5x^2 - 4x + 2, x > 0 \end{cases}$

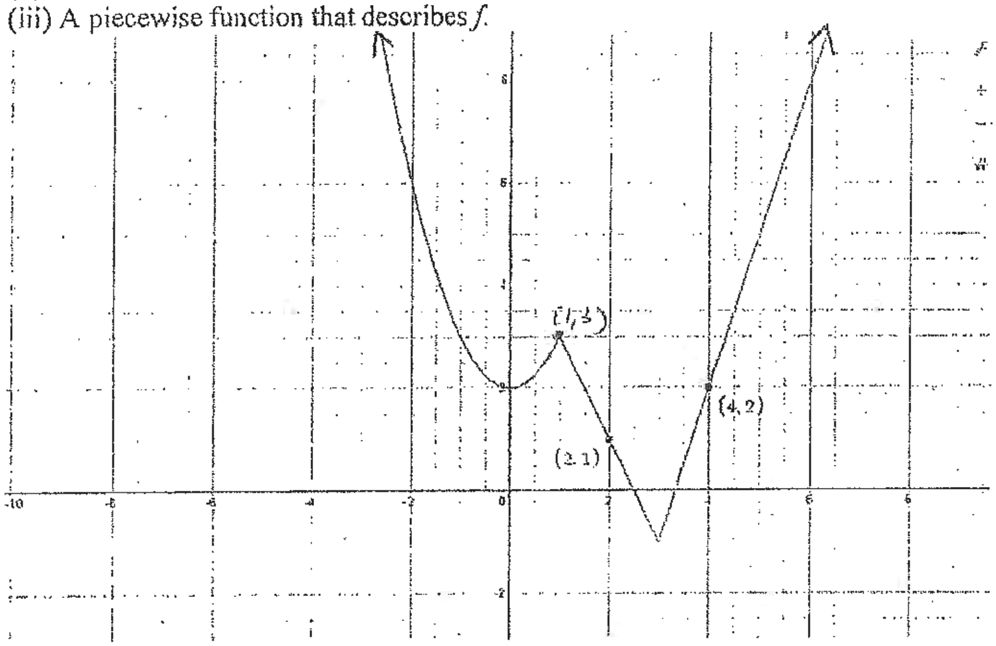
4. Given the following graph of f (consisting of a quadratic function and two line segments), find:

(i) $f(0)$, $f(-3)$, $f(f(4))$

(ii) Find where $f(x) = 2$.

(ii) The domain and range of the function

(iii) A piecewise function that describes f.



5. Identify the parent function $f(x)$ of $g(x)$ given below, and describe how the graphs of $g(x)$ and $f(x)$ are related. Then graph $f(x)$ and $g(x)$ on the same axes.

(a) $g(x) = \sqrt{x-3} + 2$ (b) $g(x) = -(x-6)^2 - 5$ (c) $g(x) = \frac{2}{x+7}$

6. For $f(x) = 4x^2 - 1$ and $g(x) = 5x - 1$, find the following functions and state their domains.

(a) $(f + g)(x)$

(b) $(f - g)(x)$

(c) $(fg)(x)$

(d) $(\frac{f}{g})(x)$

(e) $(f \circ g)(x)$

(f) $(g \circ f)(x)$

7. Find a domain on which f is one-to-one and a formula for the inverse of f restricted to this domain.

(a) $f(x) = \dfrac{1}{7x-3}$ (b) $f(x) = x^3 - 2$ (c) $f(x) = |x| + 6$

Set #2

Polynomials and Rationals:
- Divide polynomials using long division and synthetic division
- Find real and complex zeros of polynomial functions (and state multiplicity of each root)
- Graph polynomial functions
- Find end behavior of polynomials
- Analyze and graph rational functions (removable discontinuities aka "holes", vertical/horizontal asymptotes)
- Solve polynomial and rational inequalities

1. For each of the following polynomials (a) – (d) below:
(i) Describe the end behavior of the polynomial
(ii) Find the real and complex zeros of the polynomial using long division or synthetic division
(iii) Sketch the polynomial function using the information from parts (i) and (ii)

(a) $f(x) = x^3(x-3)(x+4)^2$
(b) $f(x) = -4x^3 + 24x^2 + x - 6$
(c) $f(x) = x^4 - 9x^3 + 29x^2 - 39x + 18$
(d) $f(x) = x^3 - 7x^2 + 4x - 28$

2. For each of the following rational functions (a) – (c) below:
(i) Find the domain of each function and list x- and y-intercepts
(ii) Give the coordinate location of any removable discontinuities
(iii) Give the equations of any vertical, horizontal, or slant asymptotes
(iv) Sketch the rational function using the information from parts (i) – (iii)

(a) $f(x) = \frac{x^2-16}{x^3-6x^2+5x}$ (b) $f(x) = \frac{(x-5)(x-2)}{(x-5)(x+9)}$ (c) $f(x) = \frac{x^2(x-2)(x+5)}{x^2+4x+3}$

3. Solve each inequality:
(a) $x^2 - 6x - 16 > 0$ (b) $x^3(x-3)(x+4)^2 < 0$ (c) $\frac{x^2-16}{x^3-6x^2+5x} > 0$

Set #3

Exponentials and Logarithms: Properties of exponentials and how to use them
Evaluate, analyze, and graph exponential and logarithmic functions
Converting between exponentials and logarithms
Properties of logarithms and how to use them (change of base particularly)
Solving exponential and logarithmic equations
Exponential growth and decay

1. Evaluate each expression below without using a calculator.
(a) $\log_2(\frac{1}{8})$ (b) $\ln(e^{\sqrt{2}})$ (c) $\ln(-3)$ (d) $\log_5(1)$ (e) $3^{\log_3 9}$ (f) $\log_3(\frac{1}{27})$

2. Sketch and analyze the graph of the following functions. Describe the domain, range, intercepts, asymptotes, and end behavior.
(a) $f(x) = 2^{x-3}$ (b) $g(y) = 3 + \ln(y)$ (c) $h(x) = 1 - e^x$
(d) $f(x) = \log(3x - 2)$

3. Change each logarithmic statement to an equivalent statement involving an exponent, or vice versa (depending on the situation).
(a) $a^2 = 1.6$ (b) $\log_b 4 = 2$ (c) $\ln 4 = x$

4. Write each expression as the sum and/or difference of logarithms. Express powers as factors/coefficients.
(a) $\log_3\left(\frac{uv^2}{w}\right)$, $u > 0$, $v > 0$, $w > 0$ (b) $\ln\left(x^2\sqrt{x^3+1}\right)$, $x > 0$
Write each expression as a single logarithm.
(c) $3\log_4(x^2) + \frac{1}{2}\log_4(\sqrt{x})$ (d) $2\log 2 + 3\log x - \frac{1}{2}[\log(x+3) + \log(x-2)]$
Use the Change-of-Base Formula and a calculator to evaluate each logarithm. Round your answer to three decimal places.
(e) $\log_4(19)$ (f) $\log_2(21)$

5. Solve each equation.
(a) $e^{2x} + 5 = 8$ (b) $\log_x 64 = -3$ (c) $9^{2x} = 27^{3x-4}$
(d) $\log_6(x+3) + \log_6(x+4) = 1$

6. (Sullivan, p. 331 #3) Strontium 90 is a radioactive material that decays according to the function $A(t) = A_0 e^{-0.0244t}$, where A_0 is the initial amount present and A is the amount present at time t (in years). Assume that a scientist has a sample of 500 grams of strontium 90.
(A) How much strontium 90 is left after 10 years?
(B) When will 400 grams of strontium 90 be left?
(C) What is the half-life of strontium 90?

Set #4

Sequences: Understand different types of sequences
Use sigma notation to represent and calculate sums of series
Distinguish between arithmetic and geometric sequences and find finite/infinite sums

1. Write out each sum and then find the sum of each series.

(a) $\sum_{k=1}^{10} 5$ (b) $\sum_{k=1}^{6} k$ (c) $\sum_{k=1}^{5}(5k+3)$ (d) $\sum_{k=1}^{3}(k^2+4)$

(e) $\sum_{k=1}^{6}(-1)^k 2^k$ (f) $\sum_{k=1}^{30}(k^2+4k)$

2. Express each sum using summation notation.

(a) $1+2+3+\ldots+20$ (b) $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\ldots+\frac{13}{14}$ (c) $1-\frac{1}{3}+\frac{1}{9}-\frac{1}{27}+\ldots+\frac{1}{3^6}$

3. Determine whether the given sequence is arithmetic, geometric, or neither. If the sequence is arithmetic, find the common difference; if it is geometric, find the common ratio.

(a) $\{4n^2\}$ (b) $\{3-\frac{2}{3}n\}$ (c) -1, 2, -4, 8, -16, ...

4. Find the sum of the infinite geometric series.

(a) $1+\frac{1}{3}+\frac{1}{9}+\ldots$ (b) $2-\frac{1}{2}+\frac{1}{8}-\frac{1}{32}+\ldots$ (c) $\sum_{k=1}^{\infty} 5\left(\frac{1}{4}\right)^{k-1}$

Set #5

Trigonometry: Major unit circle values
Basic trigonometric identities (Pythagorean identities)
Trigonometric graphs (sine, cosine, tangent, cotangent, secant, cosecant)
Sum and difference formulas
How to use inverse trig to find angle measures
Solve trigonometric equations using algebraic techniques.

1. Find the exact value of each expression. Do not use a calculator.

(a) $3\sin 45° - 4\tan\frac{\pi}{6}$ (b) $4\cos 60° + 3\tan\frac{\pi}{3}$ (c) $3\sin\frac{2\pi}{3} - 4\cos\frac{5\pi}{2}$

(d) $\sec^2(20°) - \tan^2(20°)$ (e) $\tan(10°)\cot(10°)$ (f) $\tan(\pi) + \sin(\pi)$

(g) $\sin(-40°)\csc(40°)$ (h) $\cos(410°)\sec(-50°)$

2. Prove each of the following identities.

(a) $\tan(\theta)\cot(\theta) - \sin^2(\theta) = \cos^2(\theta)$ (b) $\cos^2(\theta)\left(1 + \tan^2(\theta)\right) = 1$

(c) $\csc(\theta) - \sin(\theta) = \cos(\theta)\cot(\theta)$

3. For the following problem, graph each function. Each graph should contain at least one period. State the amplitude, period, and phase shift, where necessary.

(a) $y = 4\sin(2x)$ (b) $y = 3\cos(4x - \pi)$ (c) $y = 2\tan(3x)$

(d) $y = \sec(3x)$

4. Given that $\tan(\theta) = -3$, and that $\cos(\theta) < 0$, find:

(a) $\sin(2\theta)$ (b) $\cos(2\theta)$ (c) $\sin(\theta - \pi/2)$

5. Find the exact value of the expression. Do not use a calculator.

(a) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ (b) $\tan^{-1}(-\sqrt{3})$ (c) $\sec(\tan^{-1}\frac{\sqrt{3}}{3})$

6. Solve each equation on the interval $0 \le \theta \le 2\pi$.

(a) $2\cos(\theta) + \sqrt{2} = 0$ (b) $\sin(2\theta) + 1 = 0$ (c) $\sin(\theta) = \tan(\theta)$

(d) $2\cos^2(\theta) + \cos(\theta) - 1 = 0$