AP Calculus BC, Spring 2022.

Bishop McGuinness Catholic High School
Calculus BC Summer Assignment
Instructor: Mr. Paul Smith, psmith@bmhs.us

For Calculus, you will need:

(1) A 2" binder, which needs to be separate from other classes
(2) A graphing calculator

Although I list my e-mail above, I may not be available at times during the summer, so please do not expect me to always respond instantly.

There are two parts to your summer assignment.

(1) You are asked to complete three mathematical syntheses on change, limits, and functions in the space provided, along with a reflection on one of three statements made about mathematics. Please put these responses in the space provided to earn full credit. These responses will count towards your first journal entry for the course.
(2) I plan on doing a diagnostic test on AP Calculus AB material in the first ten days of class. You will find old tests and materials valuable in reviewing for this diagnostic. Don't discard them!
(3) AP Calculus BC will require us to look at the principles we learned in AP Calculus AB in a different way. To help prepare you for two of these new topics (Parametric Functions and Polar Functions), I'd like you to do some exploring on your calculator. See the last four pages for the last part of this assignment.

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I. With each synthesis, respond to and justify the principle of Calculus given below. To get full credit, it is necessary to fill all lines. Support your response with a solved example problem (perhaps from class or the book) and a visual representation (graph or table).

Principle #1: Reasoning with definitions, theorems, and properties can be used to justify claims about limits and continuity.

Solved example problem:	Visual representation:

2. With each synthesis, respond to and justify the principle of Calculus given below. To get full credit, it is necessary to fill all lines. Support your response with a solved example problem (perhaps from class or the book) and a visual representation (graph or table).

Principle #2: Derivatives allow us to solve real-world problems involving rates of change and definite integrals allow us to solve problems involving the accumulation of change over an interval.

Solved example problem:	Visual representation:

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3. With each synthesis, respond to and justify the principle of Calculus given below. To get full credit, it is necessary to fill all lines. Support your response with a solved example problem (perhaps from class or the book) and a visual representation (graph or table).

Principle #3: The Fundamental Theorem of Calculus connects differentiation and integration.

Solved example problem:

Visual representation:

4. Consider the following statements made about mathematics:

   a. "Students of calculus…say that mathematics is about rules and formulas and impossible word problems and getting the right answer by the right method."

   b. "A mathematician, like a painter or a poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with *ideas*…The mathematician's patterns, like the painter's or the poet's, must be *beautiful*; the ideas, like the colours or the words, must fit together in a harmonious way…" –Hardy

   c. "The [mathematics] problem solver goes on a long, hard backpacking trip…the problem solver gets lost, and has to find his or her way…the problem solver arrives at places of amazing beauty, and experiences ecstasy that is amplified by the effort expended to get there. When the problem solver returns home, he or she is energized by the adventure, ad cannot stop gushing about the wonderful experience." –Zeitz

   With which statement do you most agree? Use your personal mathematical experience to justify your response. (To get full credit, it is necessary to fill all the lines).

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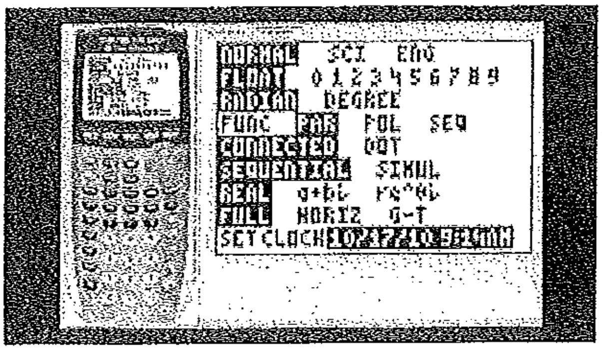
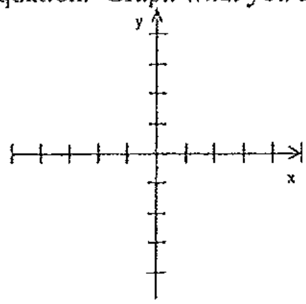
What are Parametric Functions? In these cases, the points (x, y) are not just defined on a curve, they are defined through time. See the examples below, and have your calculator ready! Where you go to find the parametric mode on your calculator is different depending on your calculator, but the picture below should help (after you go to "MODE" on your calculator)



Consider the following parametric curve: $f(t) = \begin{cases} x = 2t + 1 \\ y = 5t - 3 \end{cases}$

You'll notice that if you go to the "$y =$" button, you'll see an x_1 AND a y_1. This allows you to plug in both an x and y component to your parametric equation. Graph what you see in the space below:



We can do what is called "eliminating the parameter" to write the equation in terms of x and y (and not t). Solve for t in one of the equations and substitute it into the other equation.

When we eliminate the parameter, what equation do we get in terms of x and y?

____

What is $\frac{dy}{dx}$ for this equation? ____

How does this slope relate to $\frac{dy}{dt}$ and $\frac{dx}{dt}$? ______________________________________

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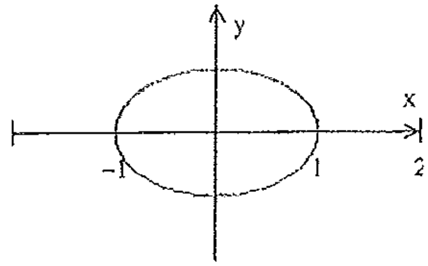
We can use our new found knowledge to find out horizontal and vertical tangents to parametric equations. Consider $f(t) = \begin{cases} x = \cos(t) \\ y = \sin(t) \end{cases}$ (The graph you get should match the graph below).



$\frac{dx}{dt} =$ ______________________ $\frac{dy}{dt} =$ ______________________

Find out which values of t where f has a horizontal tangent: ______________________

Find out which values of t where f has a vertical tangent: ______________________

How do horizontal tangents relate to the derivatives above? ______________________

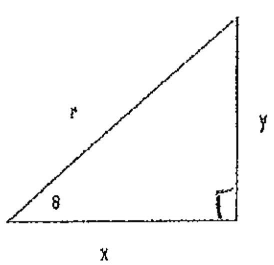
How do vertical tangents relate to the derivatives above? ______________________

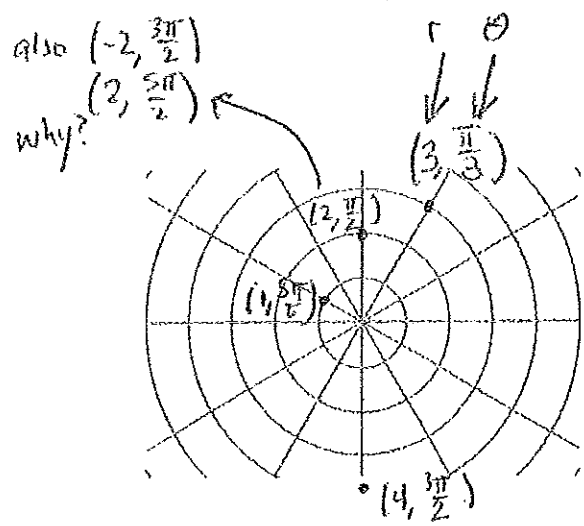
Right next to the parametric mode on the calculator is what is called "polar" mode. Instead of (x, y), we now have (r, θ).

Polar coordinates are driven by a desire to make circular objects easier to work with. For reasons you should see below, there are four major equations that we use when dealing with polar coordinates:

(1) $x = r\cos(\theta)$
(2) $y = r\sin(\theta)$
(3) $x^2 + y^2 = r^2$
(4) $\tan(\theta) = \frac{y}{x}$

r is # of radii out
θ is the angle





*Using the principles of right triangle trigonometry, explain how we get each of the 4 equations above.

(1) Since $\cos\theta = \frac{x}{r}$, $x = r\cos\theta$

(2)

(3)

(4)

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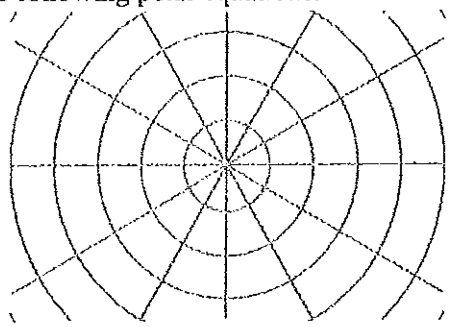
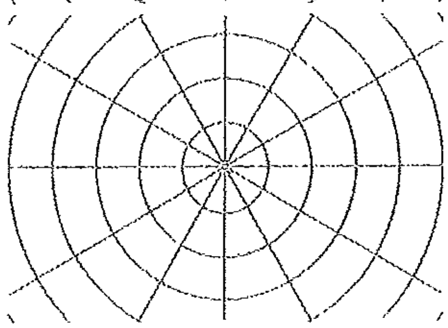
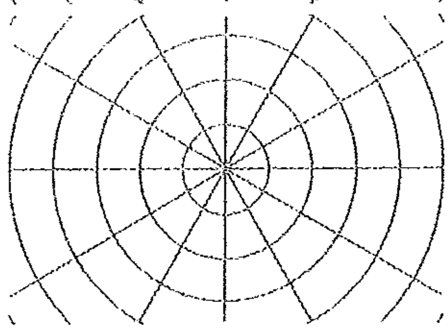
Use your calculator to sketch each of the following polar equations.



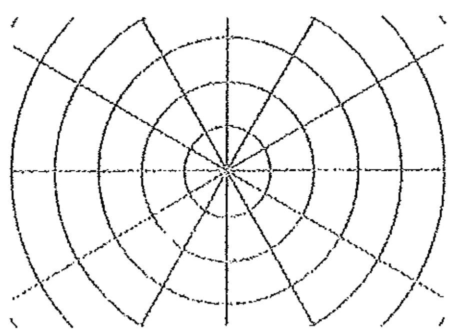
(1) $r = 2$



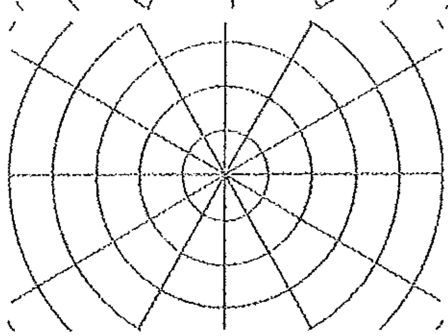
(2) $r = 4\cos(\theta)$



(3) $r = 4\sin(\theta)$



(4) $r = 3\sin(2\theta)$



(5) $r = 3\sin(3\theta)$

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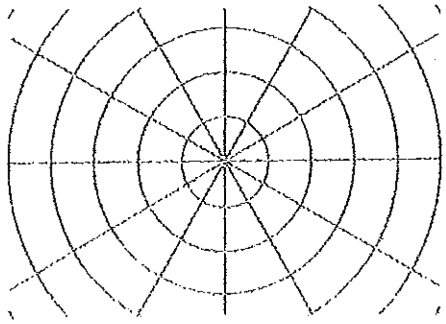
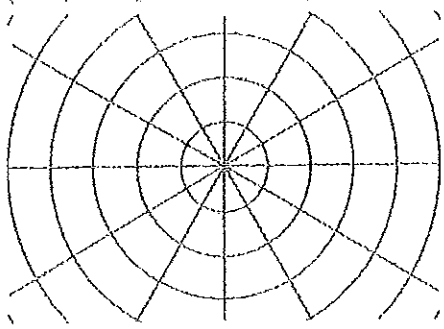
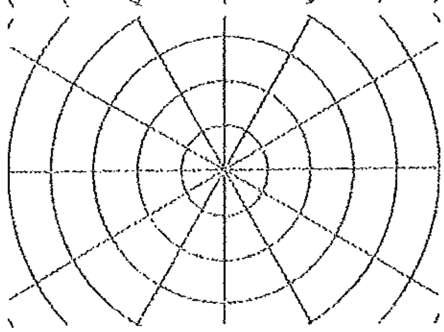


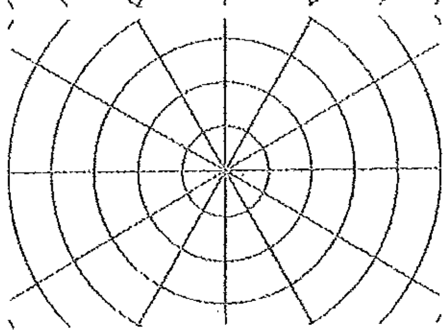
(6) $r = 1 + \sin(\theta)$



(7) $r = 1 + 2\sin(\theta)$



(8) $r = 2 + \cos(\theta)$



(9) $r = 2 + 2\cos(\theta)$

Come up with three observations you see about similarities and differences between these graphs.

1. ____

____

2. ____

____

3. ____

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